UNCLASSIFIED

CONFIDENTIAL

NDRC A-215 C.2

NATIONAL DEFENSE RESEARCH COMMITTEE

ARMOR AND ORDNANCE REPORT NO. A-215 (OSRD NO. 1801)

SECTION H and DIVISION 1

Classification cancelled or changed to:

Unclassified.

Initials of Officer Making this change

OSRD List 21 13-17 May 46 JG

INTERIOR BALLISTICS OF RECOILLESS GUNS

by

J. O. Hirschfelder, R. B. Kershner,

C. F. Curtiss and R. E. Johnson

Sep 43

20020517 113

Copy No. 65

CONFIDENTIAL

UNCLASSIFIED

UNCLASSIFIED

~~CONFIDENTIAL~~

NATIONAL DEFENSE RESEARCH COMMITTEE

ARMOR AND ORDNANCE REPORT NO. A-215 (OSRD NO. 1801)

SECTION H and DIVISION 1

INTERIOR BALLISTICS OF RECOILLESS GUNS

by

J. O. Hirschfelder, R. B. Kershner,

C. F. Curtiss and R. E. Johnson

Approved on September 7, 1943
for submission to Division 1
and Section H

R. Gibson

R. E. Gibson
Deputy Chief, Section H
Consultant, Division 1

Approved on September 8, 1943
for submission to the Committee

C. N. Hickman

C. N. Hickman, Chief
Section H

L. H. Adams

L. H. Adams, Chief
Division 1

UNCLASSIFIED

Preface

The investigation of the interior ballistics of the recoilless gun was made at the request of Section H, NDRC and Mr. S. Feltman of the Army Ordnance Department.

The authors of this report were employed under Contract OEMsr-51 -- a Division 1 contract with the Geophysical Laboratory of the Carnegie Institution of Washington.

Initial distribution of copies of the report

Nos. 1 to 24, inclusive, to the Office of the Secretary of the Committee for distribution in the usual manner;

Nos. 25 and 26 to Bureau of Ordnance (Research and Development Division);

Nos. 27 to 31 to Ordnance Department (Col. S. B. Ritchie, Maj. P. W. Constance, B. E. Anderson, S. Feltman, P. M. Netzer);

No. 32 to Watertown Arsenal (Col. H. H. Zornig);

No. 33 to Frankford Arsenal (Lt. Col. C. H. Greenall);

No. 34 to Picatinny Arsenal (Col. M. W. Kresge);

Nos. 35 and 36 to Aberdeen Proving Ground (Ballistic Research Laboratory, R. H. Kent);

No. 37 to U.S. Naval Powder Factory, Indian Head, Maryland;

No. 38 to R. C. Tolman, Vice Chairman, NDRC;

No. 39 to L. H. Adams, Chief, Division 1;

No. 40 to C. N. Hickman, Chief, Section H;

No. 41 to C. C. Lauritsen, Acting Chief, Division 3;

No. 42 to J. T. Tate, Chief, Division 6;

No. 43 to H. B. Allen, Deputy Chief, Division 1;

No. 44 to R. E. Gibson, Deputy Chief, Section H;

No. 45 to L. J. Briggs, Member, Division 1;

No. 46 to W. Bleakney, Member, Division 1;

No. 47 to E. L. Rose, Member, Division 1;

No. 48 to E. R. Weidlein, Member, Division 1;

No. 49 to W. N. Lacey, Member, Division 3;

No. 50 to J. O. Hirschfelder, Consultant, Division 3;

No. 51 to B. H. Sage, California Institute of Technology;

No. 52 to E. J. Workman, University of New Mexico;

No. 53 to R. B. Kershner, Geophysical Laboratory;
No. 54 to C. F. Curtiss, Geophysical Laboratory;
No. 55 to R. E. Johnson, Geophysical Laboratory.

The NDRC technical reports section for Armor and Ordnance edited this report and prepared it for duplication.

CONTENTS

		Page
ABSTRACT		1
Section		
1.	Introduction	1
PART I.	THE RECOILLESS GUN	
2.	Description of the gun	2
3.	The lack of recoil	2
PART II.	THE BALLISTIC EQUATIONS	
4.	Gas flow through the nozzle	3
5.	Rate of burning of the powder	4
6.	Rate of increase of powder gas in gun	4
7.	Equation of motion	5
8.	The temperature in the chamber	6
9.	The equation of state	8
10.	The ballistic equations	9
11.	The calculations of T_b and θ_{avg}	11
12.	The functions J and S	13
13.	After the powder is burned	14
PART III.	CALCULATIONAL PROCEDURE	
14.	Calculation of pressure-travel and velocity-travel curves	17
PART IV.	CHARACTERISTICS OF RECOILLESS GUNS	22
APPENDIX	List of symbols	28

List of Figures

Figure		Page
1.	Schematic diagram of a recoilless gun	2
2.	Comparison between pressure-travel curve calculated by the methods of this report and that obtained by numerical integration	23
3.	Theoretical pressure-travel and velocity-travel curves for the German 7.5-cm Leichtes Geschütz	24
4.	Pressure-travel and velocity-travel curves for a hypothetical 105-mm recoilless gun	25

INTERIOR BALLISTICS OF RECOILLESS GUNS

Abstract

A system of interior ballistics is developed for guns in which the recoil is eliminated by the rocket action of powder gas flowing through a Venturi in the breech. The system is illustrated and applied to numerous examples including the German 7.5-cm Leichtes Geschütz.

1. Introduction

In this report we present a study of the interior ballistics of recoilless guns. The equations for the interval of burning of the powder are reduced with the help of simple approximations to the equations valid for the corresponding interval in conventional guns. An approximate solution for the interval after the powder has burned is readily obtained when it is noticed that the adiabatic law holds in this case as well as in conventional guns and rockets. The accuracy of the approximations is checked by comparing a solution with the result of a numerical integration of the fundamental equations.

The ballistic system developed for recoilless guns makes possible prediction of the performance of such weapons with an accuracy that should be comparable with the results obtained by any of the leading ballistic systems for conventional guns. The methods presented here can be used to estimate the performance of any gun with a gas leakage through a vent of constant area, such as smoothbore mortars or vented mortars (where the muzzle velocity is controlled by varying the gas leakage.) In other cases of gas leakage where the vent area is not constant throughout the travel of the projectile -- such as worn guns or guns with gas operated automatic devices -- it is necessary to integrate our fundamental equations numerically.

2. Description of the gun

The recoilless gun is, in essence, a gun with an abnormally large chamber that is terminated in the rear by an open nozzle or Venturi in place of the conventional breech block (Fig. 1). Thus the recoiling forces are balanced by the rocket action of powder gases escaping to the rear. The Germans are using this type of gun at the present time,

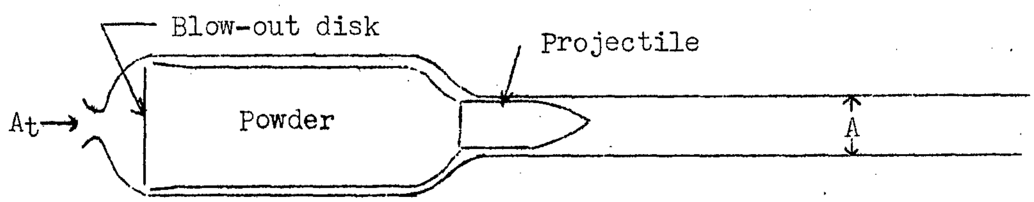

Fig. 1. Schematic diagram of a recoilless gun.

and the English are experimenting with several models. We believe that recoilless guns may well be superior to rockets and to conventional guns for a number of tactical uses. The main disadvantage of such guns is, of course, the blast of gases to the rear. This restricts their use to tactical situations in which a danger area to the rear of the gun may be kept free.

3. The lack of recoil

Fortunately, the suppression of recoil does not require a delicate balancing of forces. In fact, if the area A_t of the throat of the nozzle is the same as the cross-sectional area $\underline{A}$ of the bore and if the nozzle has no flare to the rear, then (neglecting the friction of the projectile and pressure gradients in the chamber) there is no resultant force exerted on the gun by the powder gas; hence there can be no recoil. Actually, because of the friction of the projectile, there would be a small force forward when $\underline{A}$ and A_t are equal. In addition, the gas escaping at high velocities through the nozzle creates a pressure drop along the back wall of the chamber, so that there is a further forward thrust exerted on the gun even if $A = A_t$. Thus, in order to have the

gun completely recoilless, it is necessary to design it so that $A_t < A$. The exact size of A_t will depend on the design of the nozzle, and, in particular, on the escape velocities reached by the powder gases. If the nozzle is so designed that the powder gases have a maximum velocity of the order of 7000 ft/sec, then a value of approximately 1.45 for A/A_t would be effective in eliminating the recoil. In the German 75-mm recoilless gun, the ratio A/A_t is 1.48. High escape velocities lead to an efficient gun -- that is, a small powder charge -- but high escape velocities also increase the danger zone to the rear of the gun.

PART II. THE BALLISTIC EQUATIONS

4. Gas flow through the nozzle

Let $\underline{N}$ (lb) be the weight of powder burned at any time and N' (lb) the weight of powder gas remaining in the gun at any time, so that $d(N - N')/dt$ is the time-rate of flow of the mass of gas through the nozzle. Let T_0 ($^\circ$K) be the isochoric[1/] flame temperature of the powder and $\underline{T}$ ($^\circ$K) the actual temperature of the powder gas in the chamber. Finally, let $\underline{P}$ (lb/in^2) be the average pressure in the gun at any time. Then, according to the usual engineering equation for gas flow through a nozzle,[2/]

$$d(N - N')/dt = k\sqrt{T_0/T}\, A_t P \text{ lb/sec.} \qquad (1)$$

The nozzle coefficient $\underline{k}$ may be found from the ratio of specific heats γ and the "impetus" of the powder, $F = nRT_0$ (ft lb/lb), where $\underline{n}$ is the number of moles of gas formed from unit weight of powder and $\underline{R}$ is the gas constant per mole. The equation for the nozzle coefficient $\underline{k}$ is

$$k = \sqrt{\frac{32.174\gamma}{F}\left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/(\gamma-1)}} \text{ sec}^{-1}. \qquad (2)$$

1/ Here the word isochoric is used to indicate the flame temperature at constant volume.

2/ See, for example, Stodola and Lowenstein, Steam and gas turbines (McGraw-Hill, 1927), vol. I, p. 44, Eq. (15).

For FNH-M2 powder (Hercules, 20 percent N.G.),

$$k = 0.005980 \text{ sec}^{-1} \quad [\gamma = 1.223 \text{ and } F = 384000 \text{ ft}].$$

For FNH-M1 powder (du Pont, single-base),

$$k = 0.006718 \text{ sec}^{-1} \quad [\gamma = 1.258 \text{ and } F = 310000 \text{ ft}].$$

The constants for any powder may be determined by the methods given in NDRC Report A-142.[3/]

5. Rate of burning of the powder

We suppose that the powder burns linearly at a rate which is proportional to the pressure and has a constant burning surface. Then

$$\frac{dN}{dt} = C \frac{B}{W} P \text{ lb/sec}, \tag{3}$$

where $\underline{C}$ (lb) is the total weight of the powder charge, $\underline{W}$ (in.) is the web thickness and $\underline{B}$ (in./sec)/(lb/in.^{2}) is the so-called burning constant. If the powder does not have a constant burning surface it is necessary to introduce a form function at this point.

6. Rate of increase of powder gas in gun

Combining Eqs. (1) and (3), we obtain the rate of increase of powder gas,

$$\frac{dN'}{dt} = \left[\frac{CB}{W} - k\sqrt{\frac{T_o}{T}}\, A_t\right] P \text{ lb/sec}. \tag{4}$$

We assume that the powder gas is retained in the chamber by a blow-out disk until the projectile starts to move. Let N_o be the weight of powder burned up to this time. It is convenient to define $\underline{\theta}$ by the relation,

$$\theta = \frac{\frac{CB}{W} - k\sqrt{\frac{T_o}{T}}\, A_t}{CB/W} = 1 - \frac{k\sqrt{T_o/T}\ A_t}{CB/W}. \tag{5}$$

3/ J. O. Hirschfelder, R. B. Kershner and C. R. Curtiss, Interior ballistics, I, NDRC Report A-142 (OSRD No. 1236). This report will be referred to hereafter as A-142.

It will be noticed that $\underline{\theta}$ is a linear function of $\sqrt{T_o/T}$. In terms of $\underline{\theta}$, Eq. (4) may be written,

$$\frac{dN'}{dt} = \theta \frac{CB}{W} P = \theta \frac{dN}{dt} \text{ lb/sec} \tag{6}$$

or, after integrating,

$$N' = \theta_{avg} N + (1 - \theta_{avg}) N_o \text{ lb}, \tag{7}$$

where θ_{avg} is an average value of $\underline{\theta}$ during the burning.

7. Equation of motion

The average pressure $\underline{P}$ in the gun is somewhat larger than the pressure P_X on the base of the projectile owing to the existence of a pressure gradient caused by the inertia of the powder gas. In a conventional gun,[4/]

$$P = \frac{P_X(M + C/\delta)}{M} \text{ lb/in}^2, \tag{8}$$

where $\underline{M}$ (lb) is the weight of the projectile and $\underline{\delta}$ is a parameter depending on C/M and having a value a little larger than 3 in most cases. In a recoilless gun, since the over-all momentums are balanced, there is a position of zero gas velocity in the chamber and it seems reasonable to assume that the pressure gradient established forward of this position of zero velocity is about the same as that in a conventional gun. Thus we assume that a relation similar to Eq. (8) is valid, the single difference being that only the fraction of the charge that is retained in the gun should be used. If N_o is small, then it is clear from Eq. (7) that the charge weight retained in the gun is approximately $\theta_{avg}C$. Thus we assume for a recoilless gun,

$$P = P_X(M + \theta_{avg} C/\delta)/M \text{ lb/in}^2 \tag{9}$$

It is convenient to express the equation of motion in terms of: the

---

4/ Report A-142, Eq. (2).

average pressure $\underline{P}$ (lb/in²); the velocity,

$$V = \frac{1}{12}\frac{dX}{dt} \text{ ft/sec,} \tag{10}$$

where $\underline{X}$ (in.) is the effective distance from breech to projectile defined so that AX (in³) is the total bore volume behind the projectile; and the effective mass of the projectile,

$$m' = 1.04(M + \theta_{avg} C/\delta)/32.17 \text{ slugs} \tag{11}$$

or, to a sufficiently good approximation,

$$m' = 1.04\left[M + \left(1 - \frac{kA_t}{C(B/W)}\right)\frac{C}{3}\right]\Big/32.17, \tag{11'}$$

where the factor 1.04 accounts for friction. In terms of these quantities, the equation of motion becomes

$$P = \frac{m'}{A}\frac{dV}{dt} = 12\frac{m'}{A}V\frac{dV}{dX} \text{ lb/in}^2 \tag{12}$$

8. The temperature in the chamber

The internal energy of the gas in the gun is given by the equation,

$$N'\int_0^T \underline{C_v}\, dT = N\int_0^{T_0} \underline{C_v}\, dT - \int_0^{N-N'}\int_0^T \underline{C_p}\, dT\, d(N-N') - \tfrac{1}{2}m'V^2 - \beta\tfrac{1}{2}m'V^2. \tag{13}$$

The notations $\underline{C_v}$ and $\underline{C_p}$ indicate the specific heat at constant volume and constant pressure, respectively, as functions of the temperature. Here $N\int_0^{T_0} \underline{C_v}\, dT$ is the total energy of the powder gas that is formed; $\int_0^{N-N'}\int_0^T \underline{C_p}\, dT\, d(N-N')$ is the kinetic and thermal energy of the gas that has escaped [it will be remembered that 1 lb of powder gas expanding adiabatically through a nozzle into a region of low pressure acquires the kinetic energy, $\frac{1}{2}V^2/32.17 = \int_0^T \underline{C_p}\, dT$ ft lb]; $\frac{1}{2}m'V^2$ is the kinetic energy of the projectile and of the powder gas in the gun, together with the work going into friction; and $\beta\frac{1}{2}m'V^2$ is the heat loss from the powder gas to the bore (assumed proportional to $m'V^2$).

If we set $\underline{C_p} = \underline{C_v} + nR$, let $\int_0^{T_o} \underline{C_v}\, dT = E_o$ and let C_v be the average value of $\underline{C_v}$ in the high temperature range between T_o and $\underline{T}$, then Eq. (13) becomes

$$N'[E_o - C_v(T_o - T)] = NE_o - \int_0^{N-N'} [nRT + E_o - C_v(T_o - T)]\, d(N - N') - (1+\beta)\tfrac{1}{2} m' V^2$$

$$= NE_o - (E_o - C_v T_o)(N - N') - \int_0^{N-N'} (nR + C_v)\, T\, d(N - N') - (1+\beta)\tfrac{1}{2} m' V^2 \quad (14)$$

Collecting terms and setting $\gamma = (nR + C_v)/C_v$, we obtain

$$N' C_v T = N C_v T_o - \gamma C_v \int_0^{N-N'} T\, d(N - N') - (1+\beta)\tfrac{1}{2} m' V^2 \qquad (15)$$

If $\frac{1}{2}m'V^2 = 0$ and $N_o = 0$ (so that $N' = \theta N$) the steady temperature solution of Eq. (15) is given by the equation,

$$T_o' = T_o/[\gamma - \theta(\gamma - 1)]. \qquad (16)$$

Thus, if $\theta = 1$, corresponding to a closed chamber, then $T_o' = T_o$, the isochoric flame temperature. Of if $\theta = 0$, corresponding to the steady state in a rocket motor, then $T_o' = T_o/\gamma$, the <u>isobaric</u> flame temperature.

In order to calculate the energy of the powder gas before all of the powder is burned, it is necessary to approximate the integral corresponding to the energy of the gas that is discharged. Making use of Eqs. (1) and (12), we have

$$\gamma C_v \int_0^{N-N'} T\, d(N - N') = \gamma C_v \int_0^t T \frac{d(N-N')}{dt}\, dt = \gamma C_v \int_0^t k\, A_t \sqrt{\frac{T_o}{T}}\, T P\, dt$$

$$= \gamma C_v k\, A_t \sqrt{T_o T_{avg}} \int_0^t P\, dt = \gamma C_v k \sqrt{T_o T_{avg}}\, (A_t/A) m' V. \qquad (17)$$

Here T_{avg} is a suitable average temperature in the chamber during the gas discharge. This integral has been evaluated numerically for hypothetical recoilless guns and $\sqrt{T_{avg}}$ obtained as a function of the velocity. At first $\sqrt{T_{avg}}$ equals $\sqrt{T_o}$, then drops rapidly and thereafter is linear with velocity. We make very little error if we set

$$\sqrt{T_{avg}} = \sqrt{T_o''} \left[1 - \tfrac{1}{2}(1 - \sqrt{T_b/T_o''})\, \frac{V}{V_b}\right], \qquad (18)$$

where

$$T_o'' = T_o \Big/ \left[1 + \frac{kA_t}{C(B/W)}(\gamma - 1)\right]. \tag{19}$$

Here T_b and V_b are the temperature and velocity at the time when all of the powder is burned. With this definition of $\sqrt{T_{avg}}$, Eq. (17) becomes

$$\gamma C_v \int_0^{N-N'} T\, d(N - N') = \gamma C_v k \left(\frac{A_t}{A}\right) m'V \sqrt{T_o T_o''} \left[1 - \tfrac{1}{2}(1 - \sqrt{T_b/T_o''})\, \frac{V}{V_b}\right]. \tag{20}$$

If we define $\bar{\gamma}$ as in A-142, by the relation,

$$\bar{\gamma} - 1 = (1 + \beta)(\gamma - 1), \tag{21}$$

and let

$$C_v T_o = F/(\gamma - 1). \tag{22}$$

the energy equation is expressed in terms of $\underline{T}$ and $\underline{V}$ by the relation,

$$N' \frac{T}{T_o} = N - \frac{(\bar{\gamma} - 1)\frac{1}{2}m'V^2}{F} - \gamma k \frac{A_t}{A} \sqrt{\frac{T_o''}{T_o}}\, m'V \left[1 - \tfrac{1}{2}\left(1 - \sqrt{\frac{T_b}{T_o''}}\right)\frac{V}{V_b}\right]. \tag{23}$$

9. The equation of state

Let Δ (lb/in.³), the density of the powder gas at any time, be given by the equation,

$$\Delta = \frac{N'}{AX - (C - N)/\rho}. \tag{24}$$

Here AX is the volume behind the projectile; and ρ is the density of the solid powder so that $(C - N)/\rho$ is the volume occupied by solid powder. For the equation of state of the powder gas, we use the Abel form, with the covolume η (in.³/lb),

$$12F \frac{T}{T_o} = P\left[\frac{1}{\Delta} - \eta\right] = \frac{P}{N'}\left[AX - \frac{C - N}{\rho} - \eta N'\right] \text{ in.} \tag{25}$$

Now $AX_o = v_c$ (in.³), the volume of the chamber, and $\Delta_o = C/v_c$ (lb/in.³), the initial density of loading. Thus Eq. (25) becomes

$$N'T = \frac{T_o}{12F} v_c \left[\frac{X}{X_o} - \frac{\Delta_o}{\rho} + \frac{\Delta_o}{\rho}\frac{N}{C} - \eta \Delta_o \frac{N'}{C}\right] P \;\; \text{lb}\,{}^\circ\text{K}. \tag{26}$$

In the small covolume correction term $\eta\Delta_o\frac{N'}{C}$ we approximate N' by the use of the constant θ_{avg} from Eq. (7); that is,

$$N' = \theta_{avg}N + (1 - \theta_{avg})N_o. \tag{27}$$

Also $\underline{N}$ may be found by integrating Eq. (3) with the help of Eq. (12); this gives

$$\frac{N}{C} = \frac{N_o}{C} + \frac{m'}{A}\frac{B}{W}V. \tag{28}$$

Substitution of Eqs. (27) and (28) into Eq. (26) and introduction of

$$a = \eta - (1/\rho) \tag{29}$$

and

$$j_2' = \frac{\theta_{avg}\eta - (1/\rho)}{\eta - (1/\rho)} \tag{30}$$

gives

$$N'T = \frac{T_o}{12F}v_c\left[\frac{X}{X_o} - \frac{\Delta_o}{\rho} - a\Delta_o\frac{N_o}{C} - a\Delta_o j_2'\frac{m'}{A}\frac{B}{W}V\right]P. \tag{31}$$

Here j_2' plays the same role as j_2 in A-142.

10. The ballistic equations

According to Eq. (28),

$$\frac{N}{C} = \frac{N_o}{C} + \frac{m'}{A}\frac{B}{W}V. \tag{32}$$

In particular, if V_b denotes the velocity at the instant when the powder is completely burned, that is, when $N = C$, we have

$$V_b = \frac{1 - N_o/C}{(m'/A)(B/W)}. \tag{33}$$

If we eliminate $N'T$ from Eq. (23) by the use of Eq. (31) and $\underline{N}$ by the use of Eq. (32), there results

$$\begin{aligned}\tfrac{1}{2}(\bar{\gamma} - 1)m'V^2 &+ \frac{v_c}{12}\left[\frac{X}{X_o} - \frac{\Delta_o}{\rho} - a\Delta_o\frac{N_o}{C} - a\Delta_o j_2'\frac{m'}{A}\frac{B}{W}V\right]P \\ &= CF\left[\frac{N_o}{C} + \frac{m'}{A}\frac{B}{W}V - \gamma k\frac{A_t}{AC}m'\sqrt{T_o''/T_o}\,V\right. \\ &\quad \left. + \tfrac{1}{2}\gamma k\frac{A_t}{AC}m'\sqrt{T_o''/T_o}\,(1 - \sqrt{T_b/T_o''})\frac{V^2}{V_b}\right].\end{aligned} \tag{34}$$

Now eliminating $\underline{P}$ by the equation of motion, Eq. (12), and using Eq. (33), we get the fundamental ballistic equation for the burning interval in the form of a differential equation for the velocity-travel curve, namely,

$$\tfrac{1}{2}(\bar{\gamma} - 1)m'V^2 + \frac{m'}{A}\, v_c\left[\frac{X.}{X_o} - \frac{\Delta_o}{\rho} - a\,\Delta_o\,\frac{N_o}{C} - a\,\Delta_o j_2'\,\frac{m'}{A}\,\frac{B}{W}\,V\right]V\,\frac{dV}{dX}$$

$$= CF\left[\frac{N_o}{C} + j_1'\,\frac{m'}{A}\,\frac{B}{W}\,V + k_2'\left(\frac{m'}{A}\,\frac{B}{W}\right)^2 V^2\right], \tag{35}$$

where

$$j_1' = 1 - \frac{\gamma k\, A_t\sqrt{T_o''/T_o}}{C(B/W)} \tag{36}$$

and

$$k_2' = \frac{\gamma k\, A_t\sqrt{T_o''/T_o}}{2C(B/W)}\;\frac{(1 - \sqrt{T_b/T_o''})}{(1 - N_o/C)}\,. \tag{37}$$

This equation is seen to have the same form as Eq. (26) of A-142. Following the procedure used there, we introduce

$$\alpha = \frac{\Delta_o}{\rho} + a\,\Delta_o\,\frac{N_o}{C}, \tag{38}$$

$$e_1 = \frac{CFm'(B/W)^2}{A^2}\,, \tag{39}$$

$$e_2 = \frac{CF}{A}\,\frac{B}{W}\,j_1', \tag{40}$$

$$u = \tfrac{1}{2}(\bar{\gamma} - 1) - k_2' e_1, \tag{41}$$

$$q = \frac{N_o/C}{e_1(j_1')^2}\,, \tag{42}$$

$$r = a\,\Delta_o j_1'\, j_2' e_1, \tag{43}$$

$$Z = V/e_2, \tag{44}$$

and

$$y = \frac{X}{X_o} - \alpha. \tag{45}$$

These substitutions reduce Eq. (35) to the form,

$$\frac{dy}{dZ} = \frac{Z(y - rZ)}{q + Z - uZ^2}, \tag{46}$$

which is identical with Eq. (38) of A-142.

Since the initial conditions used here are the same as in A-142, namely, $Z = 0$ when $X = X_o$, the velocity-travel relation will have the same form as Eq. (48) of A-142 -- that is,

$$X/X_o = J(1 - \alpha) - rS + \alpha + rZ, \tag{47}$$

where

$$J = \exp \int_0^Z \frac{Z\,dZ}{q + Z - uZ^2} \tag{48}$$

and

$$S = J \int_0^Z dZ/J. \tag{49}$$

The pressure can be found exactly as in A-142, by

$$P = e_3 \frac{(q + Z - uZ^2)}{J(1 - \alpha) - rS}, \tag{50}$$

where

$$e_3 = 12m' e_2^2/v_c\,. \tag{51}$$

11. The calculation of T_b and θ_{avg}

The temperature at the time when the powder is completely burned can be found by placing $V = V_b$, $N = C$, $N' = N_b'$ and $T = T_b$ in Eq. (23). This gives

$$N_b' \frac{T_b}{T_o} = C - \frac{(\gamma - 1)}{2F} m' V_b^2 - \frac{\gamma}{2} k \frac{A_t}{A} \sqrt{T_o''/T_o}\, m' V_b - \frac{\gamma}{2} k \frac{A_t}{A} m' V_b \sqrt{T_b/T_o}. \tag{52}$$

All the coefficients in this equation for $\sqrt{T_b/T_o}$ are easy to evaluate, with the exception of N_b'. The factor N_b' cannot be found easily, and we resort to an approximation method. We start with Eq. (4), which by

Eq. (12) may be written,

$$\frac{dN'}{dt} = \left[C\frac{B}{W} - k\sqrt{\frac{T_o}{T}}\,A_t\right]\frac{m'}{A}\frac{dV}{dt}. \tag{53}$$

We assume that $\sqrt{T_o/T}$ is a linear function of the velocity running from $\sqrt{T_o/T_o''}$ to $\sqrt{T_o/T_b}$ so that

$$\sqrt{T_o/T} = \sqrt{T_o/T_o''} - \left(\sqrt{T_o/T_o''} - \sqrt{T_o/T_b}\right)\frac{V}{V_b}\,. \tag{54}$$

Then, substituting this expression into Eq. (53) and integrating, we have

$$\frac{N_b'}{C} = \frac{N_o}{C} + \frac{m'}{A}\frac{B}{W}V_b - \frac{kA_t}{2C(B/W)}\left(\sqrt{T_o/T_o''} + \sqrt{T_o/T_b}\right)\frac{m'}{A}\frac{B}{W}V_b \tag{55}$$

or, making use of Eq. (33),

$$\frac{N_b'}{C} = 1 - \tfrac{1}{2}\left(1 - \frac{N_o}{C}\right)\frac{kA_t}{C(B/W)}\left(\sqrt{T_o/T_o''} + \sqrt{T_o/T_b}\right). \tag{56}$$

Since, according to Eq. (5),

$$\theta = 1 - \frac{k\sqrt{T_o/T}\,A_t}{C(B/W)}, \tag{57}$$

it is convenient to introduce

$$\theta_o'' = 1 - \frac{k\sqrt{T_o/T_o''}\,A_t}{C(B/W)}\,. \tag{58}$$

Then Eq. (55) becomes

$$\frac{N_b'}{C} = 1 - \tfrac{1}{2}\left(1 - \frac{N_o}{C}\right)(1 - \theta_o'')\left(1 + \sqrt{T_o''/T_o}\,\sqrt{T_o/T_b}\right). \tag{59}$$

Substitution of Eqs. (59) and (58) and (33) into Eq. (52) gives

$$\left[1 - \tfrac{1}{2}\left(1 - \frac{N_o}{C}\right)(1 - \theta_o'')\right]\frac{T_b}{T_o} + \left[\frac{\gamma - 1}{2}\left(1 - \frac{N_o}{C}\right)(1 - \theta_o'')\sqrt{T_o''/T_o}\right]\sqrt{T_b/T_o}$$
$$- \left[1 - \frac{\gamma}{2}\left(1 - \frac{N_o}{C}\right)(1 - \theta_o'')\frac{T_o''}{T_o} - \frac{\bar{\gamma} - 1}{2CF}m'V_b^2\right] = 0. \tag{60}$$

The temperature when the powder is completely burned is found by solving Eq. (60), a quadratic equation in $\sqrt{T_b/T_o}$.

It is found that a very satisfactory value of θ_{avg} for use in connection with Eq. (27) is found by taking

$$\theta_{avg} = 1 - \frac{k\sqrt{T_o/T_{avg}}\, A_t}{C(B/W)}, \tag{61}$$

where

$$T_{avg} = \tfrac{1}{2}(T_o'' + T_b). \tag{62}$$

12. The functions J and S

The velocity-travel relation involves the two functions J and S, given by

$$J = \exp \int_0^Z \frac{Z}{q + Z - uZ^2}\, dZ \tag{63}$$

and

$$S = J \int_0^Z (1/J)\, dZ. \tag{64}$$

Tables of these two functions covering the range that usually occurs in ballistics problems are included in A-142. However, the values of q, u and Z that occur in a typical problem of recoilless gun ballistics are outside the range of the tables of A-142. Thus it is desirable to have a convenient method for computing J and S. Since the integral defining J is elementary, the function J can be calculated accurately. However, the integral defining S is not elementary so it is necessary to resort to an approximation formula. Fortunately, an error of 5 percent in S is permissible, since S enters the ballistic equations multiplied by a coefficient which is small compared with that of J. Let

$$r_1 = (\sqrt{1 + 4uq} - 1)/(2q), \tag{65}$$

$$r_2 = (\sqrt{1 + 4uq} + 1)/(2q), \tag{66}$$

$$t_1 = 1/(r_1\sqrt{1 + 4uq}), \tag{67}$$

$$t_2 = 1/(r_2\sqrt{1 + 4uq}). \tag{68}$$

Then a convenient form of the exact formula for J is

$$J = (1 - r_1 Z)^{-t_1} (1 + r_2 Z)^{-t_2}. \tag{69}$$

An approximation formula for S usually valid to within 2 percent is

$$S = \{J + (1 - r_1 Z)^{-t_1} - (1 - r_1 Z)[1 + (1 + r_2 Z)^{-t_2}]\}/[2r_1(t_1 + 1)]. \tag{70}$$

13. After the powder is burned

(a) The temperature in the chamber. -- After all of the powder is burned, the weight N' of gas in the gun becomes a decreasing function of time while $N [= C]$ is constant. Let N_b' and T_b be the values of N' and T at the instant the powder is all burned. Then the equation of energy, Eq. (15), may be written as

$$N_b' C_v T_b - N' C_v T = A \int_{X_b}^{X} P dX - \int_{N_b'}^{N'} C_p T \, dN'. \tag{71}$$

It will be shown that this energy equation leads to the adiabatic expansion law in the case of an ideal gas. To show this we assume the equation of state of an ideal gas, that is,

$$PAX = N' nRT \text{ in. lb}, \tag{72}$$

and the adiabatic law connecting the pressure and density, that is,

$$P(AX/N')^{\gamma} = K, \tag{73}$$

where the constant K may be evaluated from

$$K = P_b (AX_b/N_b')^{\gamma} \tag{74}$$

if P_b is the pressure at the instant the powder is all burned and X_b is the value of X at the same instant. From Eqs. (72) and (73) the temperature is found as

$$T = \frac{K}{nR} \left(\frac{N'}{AX}\right)^{\gamma - 1}. \tag{75}$$

It will now be shown that Eqs. (73) and (75) provide a solution of the energy equation, Eq. (71). According to Eq. (75), the last term of

Eq. (71) becomes

$$\int_{N_b'}^{N'} C_p T\, dN' = \frac{C_p K}{\gamma nR} \int_{N_b'}^{N'} (AX)^{-(\gamma-1)}\, d(N')^{\gamma} . \tag{76}$$

Integrating by parts and using $C_p = \gamma C_v$, we obtain from Eq. (76),

$$\int_{N_b'}^{N'} C_p T\, dN' = \frac{C_v K}{nR} (AX)^{-(\gamma-1)} (N')^{\gamma} - \frac{C_v K}{nR} (AX_b)^{-(\gamma-1)} (N_b')^{\gamma}$$

$$+ (\gamma - 1)\, A \frac{C_v K}{nR} \int_{X_b}^{X} (N')^{\gamma}\, (AX)^{-\gamma}\, dX. \tag{77}$$

Substituting this value of the integral term in Eq. (71) and eliminating the temperature by Eq. (75), we get

$$N_b' C_v \frac{K}{nR} \left(\frac{N_b'}{AX_b}\right)^{\gamma-1} - N' C_v \frac{K}{nR} \left(\frac{N'}{AX}\right)^{\gamma-1} = A \int_{X_b}^{X} P\, dX$$

$$+ \frac{C_v K}{nR} (AX)^{-(\gamma-1)} (N')^{\gamma} + \frac{C_v K}{nR} (AX_b)^{-(\gamma-1)} (N_b')^{\gamma}$$

$$- (\gamma - 1)\, A \frac{C_v K}{nR} \int_{X_b}^{X} (N')^{\gamma}\, (AX)^{-\gamma}\, dX, \tag{78}$$

or

$$A \int_{X_b}^{X} P\, dX = (\gamma - 1) \frac{A C_v K}{nR} \int_{X_b}^{X} \left(\frac{N'}{AX}\right)^{\gamma} dX. \tag{79}$$

But Eq. (79) is satisfied if the pressure is given by Eq. (73), since $(\gamma - 1) C_v = nR$.

(b) Ballistic equations. -- The equation for gas flow gives

$$dN'/dt = - d(C - N')/dt = -k A_t \sqrt{T_o/T}\, P. \tag{80}$$

From the equation of motion,

$$P = \frac{m'}{A} \frac{dV}{dt} = \frac{12m'}{A} V \frac{dV}{dX}. \tag{81}$$

Finally, from Eq. (73),

$$P(AX/N')^{\gamma} = K = P_b (AX_b/N_b')^{\gamma} . \tag{82}$$

If we assume that the temperature $\underline{T}$ occurring in Eq. (80) is constant, then the ballistic equations for the interval after the powder is burned may be obtained from Eqs. (80), (81) and (82).

In Eq. (80), let $\sqrt{T_o/T} = \sqrt{T_o/T_b}$ = const. Then, from Eqs. (80) and (81),

$$\frac{dN'}{dt} = -k \frac{A_t}{A} \sqrt{T_o/T_b}\, m' \frac{dV}{dt} \tag{83}$$

or, integrating,

$$N' = N_b' - \frac{km'\sqrt{T_o/T_b}}{A/A_t} (V - V_b). \tag{84}$$

Let

$$\phi = \frac{V_b k m' \sqrt{T_o/T_b}}{N_b' (A/A_t)} \tag{85}$$

and

$$v = V/V_b. \tag{86}$$

Then Eq. (84) may be written,

$$N' = N_b'[1 + \phi - \phi v]. \tag{87}$$

Now $\underline{P}$ and N' may be eliminated from Eq. (82) by the use of Eqs. (81) and (87). This gives

$$\frac{12m'}{A} V \frac{dV}{dX}\left[\frac{AX}{N_b'(1 + \phi - \phi v)}\right]^{\gamma} = P_b\left[\frac{AX_b}{N_b'}\right]^{\gamma}. \tag{88}$$

Using Eq. (86) and introducing

$$Y = X/X_b, \tag{89}$$

$$\Omega = A P_b X_b \phi^{\gamma}/12\, m' V_b^2, \tag{90}$$

and

$$\Psi = \frac{1}{\phi} + 1, \tag{91}$$

we may write Eq. (88) as

$$v \frac{dv}{dY}\left(\frac{Y}{\Psi - v}\right)^{\gamma} = \Omega. \tag{92}$$

Eq. (92) may be integrated by separating variables. The result is

$$\Omega \int_1^Y \frac{dY}{Y^\gamma} = \int_1^v \frac{v\,dv}{(\Psi - v)^\gamma}, \tag{93}$$

or

$$\Omega(2 - \gamma)\left[1 - \frac{1}{Y^{\gamma-1}}\right] = \frac{\Psi - (\gamma - 1)v}{(\Psi - v)^{\gamma-1}} - \frac{\Psi - (\gamma - 1)}{(\Psi - 1)^{\gamma-1}}. \tag{94}$$

Solution of Eq. (94) for $Y^{\gamma-1}$ gives

$$Y^{\gamma-1} = \frac{\Omega(2 - \gamma)}{\Omega(2-\gamma) + \phi^{\gamma-2}[1 + (2-\gamma)\phi] - \frac{[\Psi - (\gamma - 1)v]}{(\Psi - v)^{\gamma-1}}}. \tag{95}$$

This equation gives the velocity-travel relation for the interval after the powder is burned.

The pressure may be easily found as a function of the velocity and travel. From the equation of motion, Eq. (81), and Eqs. (86), (89) and (90), it follows that

$$P = \frac{12m'}{A} V \frac{dV}{dX} = \frac{12m'V_b^2}{AX_b} v \frac{dv}{dY} = \frac{P_b \phi^\gamma}{\Omega} v \frac{dv}{dY}. \tag{96}$$

Combination of Eq. (92) and Eq. (96) gives

$$P = P_b\left[\frac{\phi(\Psi - v)}{Y}\right]^\gamma. \tag{97}$$

Thus the travel and pressure corresponding to any preassigned velocity may be found from Eq. (95) and Eq. (97), respectively. Unfortunately, it is not possible to express the velocity in terms of the travel conveniently so that muzzle velocities must be found by successive approximations.

PART III. CALCULATIONAL PROCEDURE

14. Calculation of pressure-travel and velocity-travel curves

To illustrate the method developed in this report, we calculate the pressure-travel and velocity-travel curves for a hypothetical recoilless gun.

The gun constants are assumed to be as follows:

D	**Diameter of bore**	4.134 in. (105 mm)
A	Area of bore	13.7 in^2
A_t	Area of throat	9.45 in^2
v_c	Volume of chamber	356.9 in^3
L_m	Travel to muzzle	80 in.
P_o	Starting pressure	5000 lb/in^2

The powder constants are:

F	Impetus	384000 ft lb/lb
γ	Ratio of specific heats	1.223
η	Covolume	26.95 in^3/lb
ρ	Density of solid powder	0.0596 lb/in^3
k	Nozzle coefficient	0.00598 sec^{-1}
B	Burning constant	0.00055 (in./sec)/(lb/in^2)

The loading constants are:

M	Weight of projectile	29.76 lb
W	Web thickness of powder grains	0.0514 in.
C	Weight of powder charge	8.379 lb

First calculate the following preliminary quantities:

Quantity	Numerical Value
$X_o = v_c/A$	26.05
$\Delta_o = C/v_c$	0.02348
$a = \eta - 1/\rho$	10.17
$j_o = \dfrac{P_o(1 - \Delta_o/\rho)}{\Delta_o(12F + aP_o)}$	0.02769
β	0.4
$\lambda = \dfrac{kA_t}{C(B/W)}$	0.6303
$\Gamma = \sqrt{(\gamma - 1)\lambda + 1}$	1.0680
$m = \dfrac{M + (1 - \lambda)C/3}{32.174}$	0.9570
$m' = 1.04m$	0.9953

Quantity	Numerical Value
$\bar{\gamma} = (1 + \beta)(\gamma - 1) + 1$	1.3122
$V_b = \dfrac{(1 - j_o)A}{m'(B/W)}$	1250
$E_b = (\bar{\gamma} - 1)m'V_b^2/2\,CF$	0.07545
$g_o = \frac{1}{2}(1 - j_o)\lambda$	0.3064
$g_1 = 1 - g_o\Gamma$	0.6728
$g_2 = g_o(\gamma - 1)/2$	0.03416
$g_3 = 1 - E_b - g_o\gamma/\Gamma$	0.5737
$\dfrac{T_b}{T_o} = \left(\dfrac{-g_2 + \sqrt{g_2^2 + g_1 g_3}}{g_1}\right)^2$	0.7642
$\zeta = \sqrt{\dfrac{1}{2\Gamma^2} + \frac{1}{2}\cdot\dfrac{T_b}{T_o}}$	0.9058
$\theta_{avg} = 1 - \lambda/\zeta$	0.3042
$j_1' = 1 - \gamma\lambda/\Gamma$	0.2782
$j_2' = \dfrac{\eta\theta_{avg} - 1/\rho}{a}$	-0.8438
$k_2' = \dfrac{(1 - j_1')(1 - \Gamma\sqrt{T_b/T_o})}{2(1 - j_o)}$	0.02463
$e_1 = \dfrac{CF m'(B/W)^2}{A^2}$	1.9534
$e_2 = \dfrac{CF(B/W)\,j_1'}{A}$	699.1
$e_3 = 12m'e_2^2/v_c$	16,355
$r = a\Delta_o j_1' j_2' e_1$	-0.1095
$\alpha = \Delta_o/\rho + a\Delta_o j_o$	0.4006
$q = j_o/(j_1')^2 e_1$	0.1832
$u = \frac{1}{2}(\bar{\gamma} - 1) - k_2' e_1$	0.1080
$Z_b = V_b/e_2$	1.7880

Corresponding to any value of the parameter $\underline{Z}$ between zero and Z_b there is a travel given by the equation,

$$L_X = X_o\left(\frac{X}{X_o} - 1\right),$$

where[5/]

$$\frac{X}{X_o} = J(1 - \alpha) - rS + \alpha + rZ.$$

The pressure and velocity for this travel are given by

$$P = \frac{e_3(q + Z - uZ^2)}{J(1 - \alpha) - rS}$$

and

$$V = e_2 Z.$$

The **maximum pressure** P_p occurs at Z_p,

$$Z_p = \left[1 + \frac{rP_p}{e_3}\right] / (1 + 2u)$$

or Z_b, whichever is smaller. It will be observed that P_p, the maximum pressure, is used to obtain a value of Z_p. However, rP_p/e_3 is small in comparison to unity, and thus a sufficiently accurate value of P_p can be found by a method of successive approximations in two or three steps.

The method of finding X_b/X_o and P_b will now be given in detail. The calculations for any other point would be the same as those given below, except that Z_b would be replaced by the value of $\underline{Z}$ at the chosen point.

	Quantity	Numerical Value
r_1	$= (\sqrt{1 + 4uq} - 1)/2q$	0.10594
r_2	$= (\sqrt{1 + 4uq} + 1)/2q$	5.5645
t_1	$= 1/(r_1\sqrt{1 + 4uq})$	9.0866
t_2	$= 1/(r_2\sqrt{1 + 4uq})$	0.17300
J	$= (1 - r_1 Z_b)^{-t_1} (1 + r_2 Z_b)^{-t_2}$	4.4557
S	$= \{J + (1 - r_1 Z_b)^{-t_1} - (1 - r_1 Z_b)[1 + (1 + r_2 Z_b)^{-t_2}]\}/[2r_1(t_1 + 1)]$	4.609

5/ $\underline{J}$ and $\underline{S}$ are the functions of $\underline{q}$, $\underline{u}$ and $\underline{Z}$ defined in Sec. 12.

Quantity	Numerical Value
$\frac{X_b}{X_o} = J(1-\alpha) - rS + \alpha + rZ_b$	3.3802
$P_b = \frac{e_3(q + Z_b - uZ_b^2)}{J(1-\alpha) - rS}$	8374
$L_{X_b} = X_o\left(\frac{X_b}{X_o} - 1\right)$	62.00
$V_b = e_2 Z_b$	1250

The calculations **after** the powder is all burned are as follows:

Quantity	Numerical Value
$N_b'/C = g_1 - g_o/\sqrt{T_b/T_o}$	0.3223
$\phi = \frac{2(g_1 - N_b'/C)}{N_b'/C}$	2.175
$\Psi = 1/\phi + 1$	1.4598
$\xi = \frac{e_3 Z_b^2}{P_b \phi^{\bar{\gamma}} (2 - \bar{\gamma})(X_b/X_o)}$	0.9688

The travel corresponding to any velocity $\underline{V}$ greater than V_b is given by

$$L = X_o\left(Y \frac{X_b}{X_o} - 1\right),$$

where

$$Y = \left\{1 + [1 + (2-\bar{\gamma})\phi]\,\phi^{\bar{\gamma}-2}\xi - \frac{\xi[\Psi - (\bar{\gamma} - 1)V/V_b]}{(\Psi - V/V_b)^{\bar{\gamma}-1}}\right\}^{-\frac{1}{\bar{\gamma}-1}}.$$

The corresponding pressure is given by

$$P = P_b\left[\frac{\phi(\Psi - V/V_b)}{Y}\right]^{\bar{\gamma}}$$

The velocity corresponding to any travel after the powder is all burned cannot be found directly. The method used here is to calculate the travel at several velocities, and then interpolate to find the

velocity at the muzzle. The solution to our example corresponds to taking V = 1355 ft/sec, Y = 1.2095 and L = 80.4 in. The corresponding pressure $\underline{P}$ is 5010 lb/in^2.

The pressure-travel and velocity-travel curves for this example are given in Fig. 2. There are small errors in these curves introduced by the mathematical approximations explained in the text. The magnitude of these errors is shown by a comparison with the dotted curve in Fig. 2. This dotted curve was obtained by a point-to-point numerical integration of the fundamental equations. The velocity-travel curves are indistinguishable on this graph. It is seen that the mathematical errors introduced by our approximations are negligible compared to the probable physical uncertainties.

PART IV. CHARACTERISTICS OF RECOILLESS GUNS

The only existing recoilless gun that we have considered is the German 7.5-cm L.G. Its characteristics are shown in Table I and the pressure-travel and velocity-travel curves that we calculate for it are shown in Fig. 3. This gun operates at relatively high pressures and therefore requires a comparatively short travel.

Table I. Characteristics of the German 7.5-cm Leichtes Geschütz.

Chamber volume, v_c	120 in^3
Density of loading, Δ_o	0.63 gm/cm^3
Ratio A/A_t	1.45
Maximum pressure, P_p	29000 lb/in^2
Muzzle velocity, V_m	1210 ft/sec
Weight of powder, C	2.75 lb
Approximate web, W	0.02 in.
Muzzle pressure, P_m	9000 lb/in^2
Weight of gun in action	320 lb

In order to examine the possible characteristics of recoilless guns, we made a large number of calculations on hypothetical 105-mm recoilless guns having various ballistic parameters. These characteristics are shown in Table II. The following conclusions may be drawn.

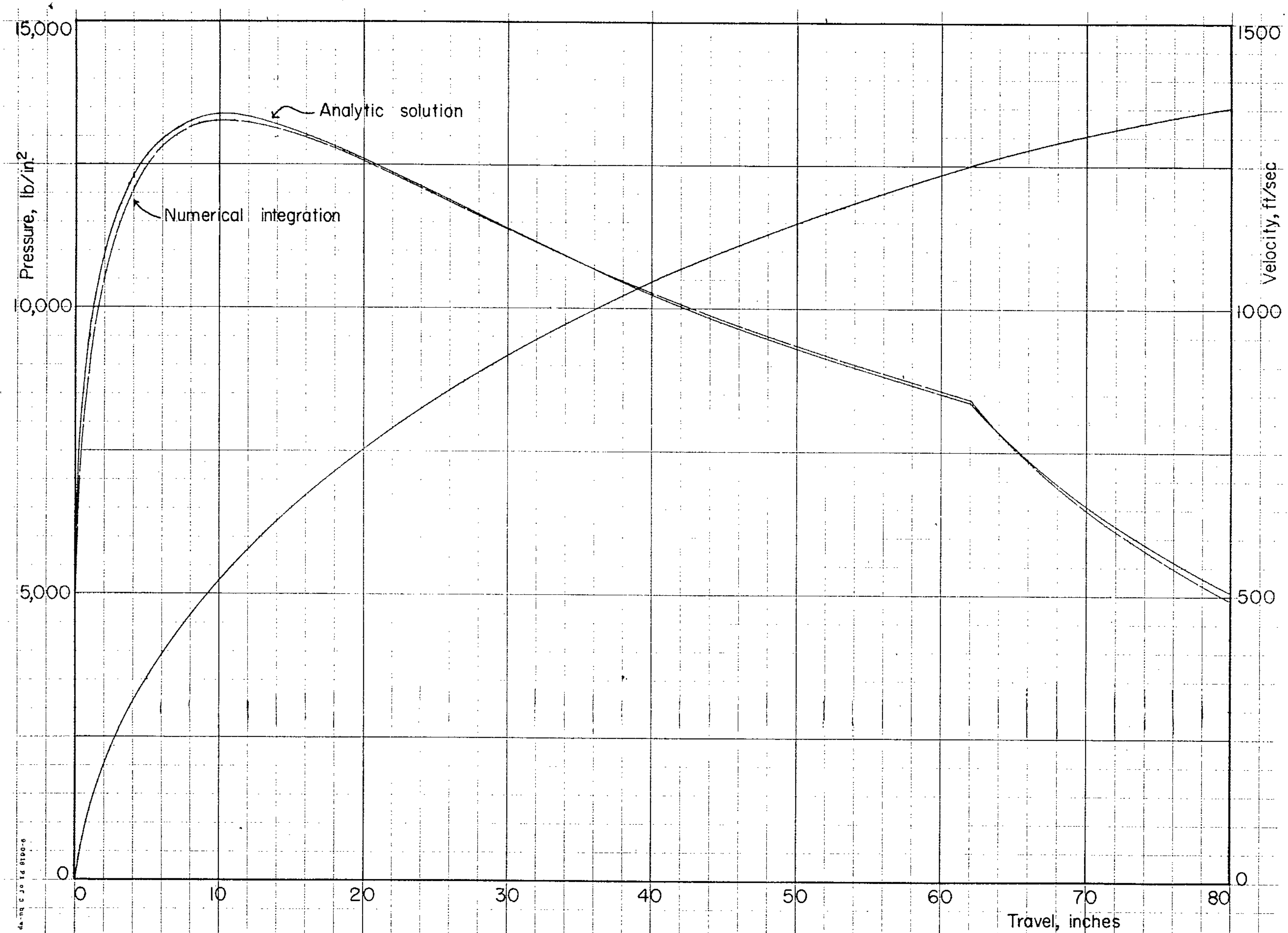

Fig. 2 Comparison between pressure-travel curve calculated by the methods of this report and that obtained by numerical integration.

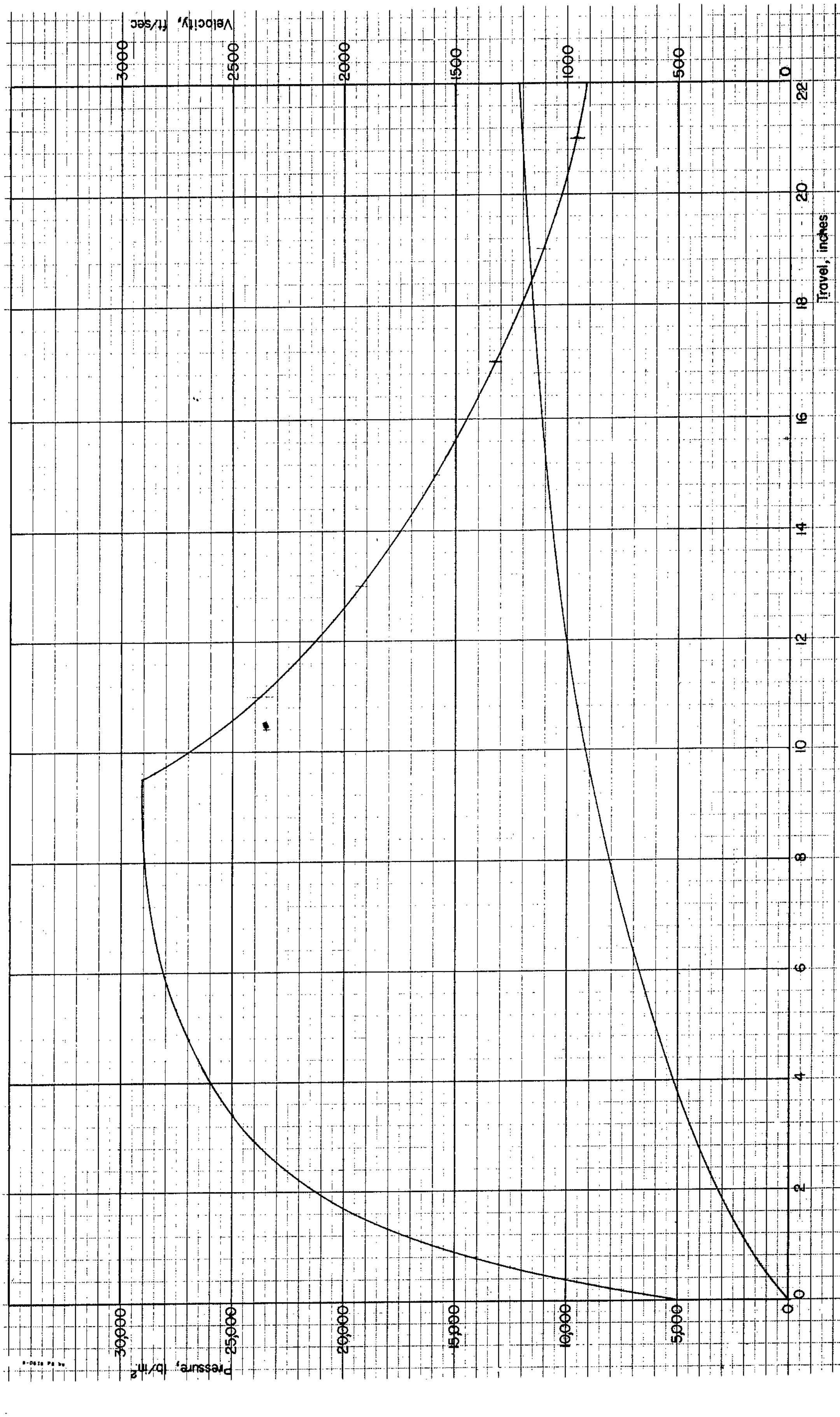

Fig 3. Theoretical pressure-travel and velocity-travel curves for the German 7.5-cm Leichtes Geschütz

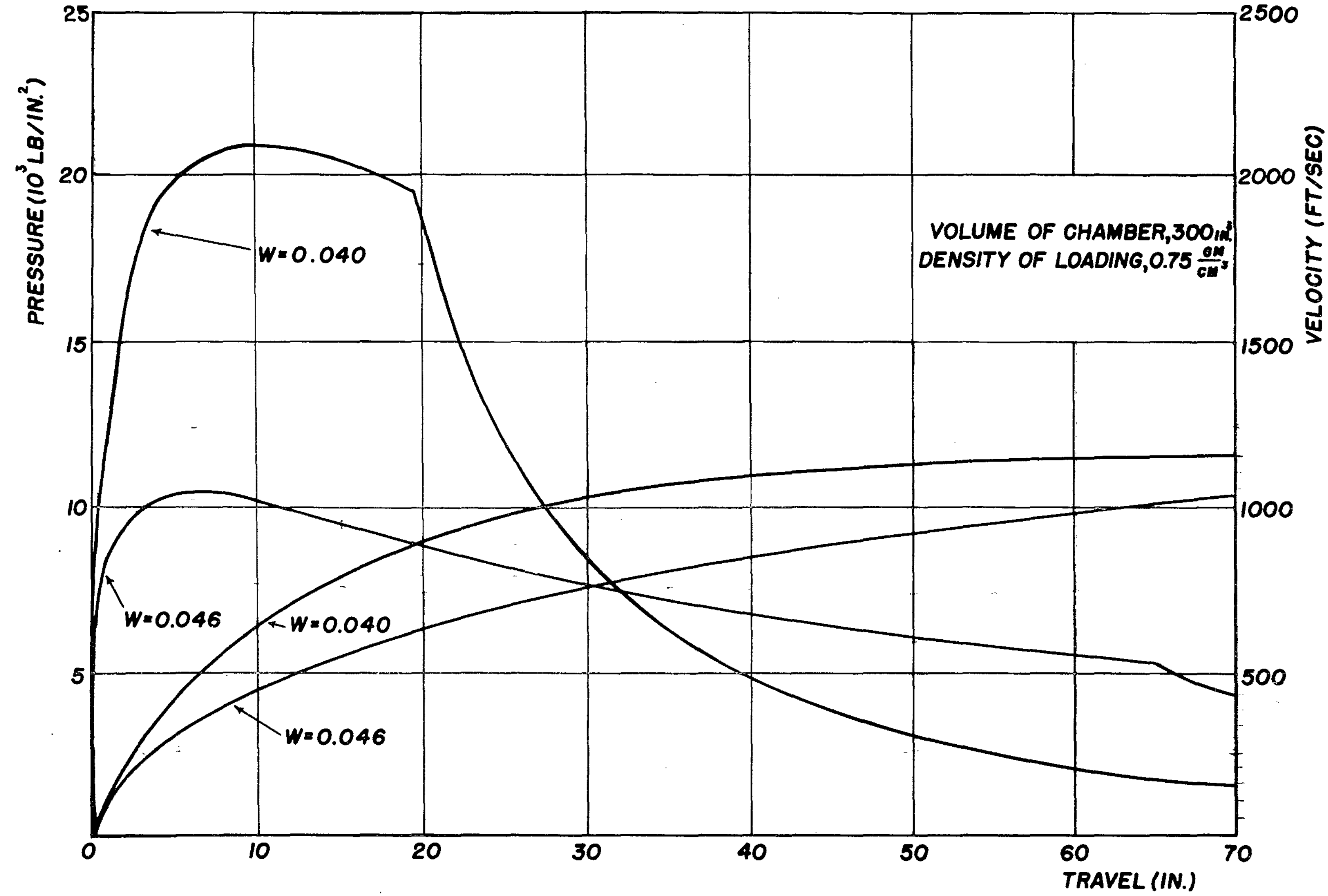

FIG. 4. PRESSURE-TRAVEL AND VELOCITY-TRAVEL CURVES FOR A HYPOTHETICAL 105-MM RECOILLESS GUN, THE EFFECT OF VARYING THE WEB W IS ILLUSTRATED.

Table II. Calculations for hypothetical 105-mm recoilless guns (L = 70 in., M = 33 lb, FNH-M2 powder used).

Chamber Volume, v_c (in^3)	Density of Loading (gm/cm^3)	Ratio A/A_t	Maximum Pressure, P_p (lb/in^2)	Muzzle Velocity, V_m (ft/sec)	Weight of Powder, $\underline{C}$ (lb)	Approximate Web, $\underline{W}$ (in.)	Muzzle Pressure, P_m (lb/in^2)	Starting Pressure, P_o (lb/in^2)	Blow-out Pressure (lb/in^2)
300	0.65	1.16	9000	950	7.04	0.041	3000	5000	5000
300	.65	1.16	19500	1040	7.04	.035	1000	5000	5000
300	.75	1.16	10500	1050	8.13	.046	4300	5000	5000
300	.75	1.16	21000	1170	8.13	.040	1500	5000	5000
400	.65	1.16	12000	1150	9.39	.050	5700	5000	5000
400	.65	1.16	20000	1280	9.39	.045	3000	5000	5000
400	.65	1.16	42000	1330	9.39	.037	--	5000	5000
400	.75	1.16	14000	1270	10.84	.056	7800	5000	5000
400	.75	1.16	19000	1390	10.84	.053	4800	5000	5000
400	.75	1.45	17000	1400	10.84	.064	10400	5000	5000
587	.65	1.16	19000	1510	13.78	.063	9500	5000	5000
587	.65	1.45	19800	1590	13.78	.072	14000	5000	5000
400	.65	1.16	13400	1210	9.39	.046	4100	1000	1000
400	.65	1.16	17700	1260	9.39	.046	3000	5000	5000
400	.65	1.16	21200	1290	9.39	.046	2500	5000	10000
400	.65	1.16	22300	1290	9.39	.046	2600	10000	10000

(i) The muzzle velocity is quite insensitive to the burning rate or web of the powder; consequently, it is insensitive to the maximum pressure. It is also quite insensitive to the starting pressure.

(ii) The maximum pressure is very sensitive to the burning rate or the web of the powder; as a consequence, the maximum pressure is very sensitive to the initial temperature of the powder. This shortcoming is reminiscent of the corresponding problem in conventional rockets.

(iii) It is possible to design a recoilless gun operating at low pressures that has a very flat pressure-travel curve. This is not feasible for recoilless guns operating at high pressures since the density of loading cannot be increased sufficiently. This is shown in Fig. 4.

(iv) The muzzle velocity is not sensitive to the cross-sectional area of the throat provided the web of the powder is adjusted so as to give approximately the same maximum pressure.

(v) It is desirable to use a very high density of loading in recoilless guns.

It seems probable that most of the tactical uses of rockets can be served by recoilless guns. The weight of the projector and a few rounds of ammunition is quite comparable in the two cases since the recoilless feature makes possible a very light mount. In addition, the recoilless gun has the definite advantage that it fires conventional projectiles with good exterior ballistic characteristics. The accuracy that is claimed for existing recoilless guns is quite comparable to that for conventional howitzers.

There are several problems that must be met in the design of a satisfactory recoilless gun. First, a satisfactory method for trapping the powder must be found if the velocity dispersion is to be kept small. Second, it is desirable to design a blow-out disk that holds to a fairly high pressure and then shatters into small fragments which do little damage. Third, it is desirable to reduce the tremendous blast that results from the escape of powder gas to the rear of the gun. The blast in the German 7.5-cm gun is so severe that the personnel must use special ear protectors.

APPENDIX

List of Symbols

A	$in.^2$	Cross-sectional area of the bore.
A_t	$in.^2$	Area of the throat.
a	$in.^3/lb$	$\eta - (1/\rho)$.
B	$(in./sec)/(lb/in.^2)$	Burning constant.
C	lb	Total weight of the powder charge.
C_p	ft lb/lb °K	Specific heat at constant pressure of the powder gas; $C_p = C_v + nR$. When a function of temperature, denoted by $\underline{C_p}$.
C_v	ft lb/lb °K	Specific heat at constant volume of the powder gas. When a function of temperature, denoted by $\underline{C_v}$.
D	in.	Diameter of the bore.
E_o	ft lb/lb	$\int_0^{T_o} \underline{C_v}\, dT$.
E_b	---	$(\bar{\gamma} - 1)m'V_b^2/2CF$.
e_1	---	$CFm'(B/W)^2/A^2$.
e_2	ft/sec	$(CF/A)(B/W)\,j_1'$.
e_3	$lb/in.^2$	$12m'e_2^2/v_c$.
F	ft lb/lb	Impetus of the powder, equal to nRT_o .
g_o	---	$\frac{1}{2}(1 - j_o)\lambda$.
g_1	---	$1 - g_o\Gamma$.
g_2	---	$\frac{1}{2}g_o(\gamma - 1)$.
g_3	---	$1 - E_b - g_o\gamma/\Gamma$.
J	---	$\exp \int_0^Z \frac{Z\,dZ}{q + Z - uZ^2}$.
j_1'	---	$1 - \frac{\gamma k A_t \sqrt{T_o''/T_o}}{C(B/W)}$

j_2'	---	$\dfrac{\theta_{avg}\eta - (1/\rho)}{\eta - (1/\rho)}$.
j_o	---	$\dfrac{P_o(1 - \Delta_o/\rho)}{\Delta_o(12F + a P_o)}$.
K	---	$P_b(AX_b/N_b')^\gamma$.
k	sec^{-1}	Nozzle coefficient; $k = \left[\dfrac{32.174\gamma}{F}\left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{\frac{1}{2}}$.
k_2'	---	$\dfrac{\gamma k A_t \sqrt{T_o''/T_o}}{2C(B/W)} \; \dfrac{(1 - \sqrt{T_b/T_o''})}{(1 - N_o/C)}$.
L	in.	Total travel of the projectile.
M	lb	Weight of the projectile.
m	slug	$\dfrac{M + (1 - \lambda)C/3}{32.174}$.
m'	slug	Effective mass of the projectile, $m' = 1.04(M + \theta_{avg}\, C/\delta)/g = 1.04m$.
N	lb	Weight of powder burned at any time.
N'	lb	Weight of powder gas remaining in the gun at any time.
N_b'	lb	Value of N' at the instant the powder is all burned.
N_o	lb	Powder burned before projectile starts to move.
n	mole/lb	Number of moles of gas formed from unit weight of powder.
P	lb/in^2	Average pressure in the gun at any time.
P_b	lb/in^2	Pressure at the instant the powder is all burned.
P_o	lb/in^2	Starting pressure.
P_p	lb/in^2	Maximum pressure.
P_x	lb/in^2	Pressure on the base of the projectile.

q	---	$(N_o/C)/e_1(j_1')^2$.
R	ft lb/(mole °K)	Gas constant per mole.
r	---	$a\Delta_o j_1' j_2' e_1$.
r_1	---	$(\sqrt{1+4uq}-1)/2q$.
r_2	---	$(\sqrt{1+4uq}+1)/2q$.
S	---	$J\int_0^Z dZ/J$.
T	°K	Temperature of the powder gas in the chamber.
T_{avg}	°K	Average temperature in the chamber during the gas discharge. Assumed equal to $\frac{1}{2}(T_o'' + T_b)$.
T_b	°K	Value of $\underline{T}$ at the instant the powder is all burned.
T_o	°K	Isochoric flame temperature of the powder.
T_o'	°K	$T_o/[\gamma - \theta(\gamma - 1)]$.
T_o''	°K	$T_o\Big/\left[1 + \frac{k A_t}{C B/W}(\gamma - 1)\right]$.
t_1	---	$1/r_1\sqrt{1+4uq}$.
t_2	---	$1/r_2\sqrt{1+4uq}$.
u	---	$\frac{1}{2}(\bar{\gamma} - 1) - k_2' e_1$.
V	ft/sec	Velocity of the projectile at any time.
V_b	ft/sec	Velocity at the time when all powder is burned.
v	---	V/V_b .
v_c	in.³	Volume of the chamber, equal to AX_o .
W	in.	Web thickness of the powder grains.
X	in.	Effective distance from the breech to the projectile such that AX is the volume behind the projectile.

X_o	in.	Effective length of the powder chamber, equal to v_c/A.
Y	---	X/X_b.
y	---	$(X/X_o) - \alpha$.
Z	---	V/e_2.
Z_b	---	Value of $\underline{Z}$ at which maximum pressure occurs.
α	---	$(\Delta_o/\rho) + a\Delta_o N_o/C$.
β	---	Ratio of heat loss to kinetic energy of projectile at muzzle.
Γ	---	$\sqrt{(\gamma - 1)\lambda + 1}$.
γ	---	Ratio of specific heats; $\gamma = (nR + C_v)/C_v = 1 + 1.9869\ n/C_v$.
$\bar{\gamma}$	---	Pseudo ratio of specific heats; $\bar{\gamma} = 1 + (1 + \beta)(\gamma - 1)$.
Δ	lb/in.^{3}	Density of the powder gas at any time; $\Delta = N'/[AX - (C - N)/\rho]$.
Δ_o	lb/in.^{3}	Initial density of loading, equal to C/v_c.
δ	---	Parameter depending on C/M and usually having a value a little larger than 3; see A-142.
ζ	---	$\left[\frac{1}{2\Gamma^2} + \frac{1}{2}\frac{T_b}{T_o}\right]^{\frac{1}{2}}$.
η	in.^{3}/lb	Covolume in the equation of state for the powder gas.
θ	---	$1 - \frac{k\sqrt{T_o/T}\,A_t}{C(B/W)}$.
θ_{avg}	---	Average value of θ during the burning; $\theta_{avg} = 1 - \frac{k\sqrt{T_o/T_{avg}}\,A_t}{C(B/W)}$.
θ''_o	---	$1 - \frac{k\sqrt{T_o/T''_o}\,A_t}{C(B/W)}$.
λ	---	$kA_t/C(B/W)$.

ξ	---	$\dfrac{e_3 Z_b^2}{P_b \phi^\gamma (2-\gamma)(X_b/X_o)}$.
ρ	$lb/in.^3$	Density of the solid powder.
ϕ	---	$\dfrac{V_b k m' \sqrt{T_o/T_b}}{N_b' (A/A_t)}$.
Ψ	---	$(1/\phi) + 1$.
Ω	---	$\Delta P_b X_b \phi^\gamma / 12 m' V_b^2$.

[redacted]

TITLE: Interior Ballistics of Recoilless Guns

ATI- 23617
DIVISION (None)
ORIG. AGENCY NO. A-215
PUBLISHING AGENCY NO. OSRD 1801

AUTHOR(S): Hirschfelder, J. O.; Kershner, R. B.; Curtiss, C. F. and others
ORIGINATING AGENCY: Carnegie Institution of Washington
PUBLISHED BY: Office of Scientific Research and Development, Div. 3, Washington, D. C.

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
Sept '43	[redacted]	U.S.	Eng.	37	table, diagr, graphs

ABSTRACT:

A system of interior ballistics is developed for guns in which the recoil is eliminated by the rocket action of the powder gas flowing through a venturi in the breech. Ballistic equations are derived on the basis of simplifying assumptions, and the accuracy of the approximations is checked by comparing a solution with the results of a numerical integration of the fundamental equations. The system is illustrated and applied to numerous examples including the German 7.5-cm Leichtes Geschuetz.

DISTRIBUTION: Copies of this report obtainable from Air Documents Division; Attn: MCIDXD

DIVISION: Ordnance and Armament (22)
SECTION: Guns (2)

ATI SHEET NO.: C-22-2-4

SUBJECT HEADINGS: Guns, Recoilless - Interior ballistics (47476.7)

[redacted]

TITLE: Interior Ballistics of Recoilless Guns			ATI- 25028
			REVISION (None)
AUTHOR(S): Hirschfelder, J. O.; Kershner, R. B.; Curtiss, C. F. and others			ORIG. AGENCY NO. (None)
ORIGINATING AGENCY: Carnegie Institution of Washington, Washington, D. C.			
PUBLISHED BY: Office of Scientific Research and Development, NDRC, Div. 1			PUBLISHING AGENCY NO. OSRD-1801

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
Sept '43	Unclass.	U.S.	Eng.	[illegible]	tables, graphs

ABSTRACT:

A system of interior ballistics is developed for guns in which the recoil is eliminated by the rocket action of powder gas flowing through a venturi in the breech. The system is illustrated and applied to numerous examples including the German 7.5 cm Leichtes Geschuetz. This system makes possible prediction of the performance of such weapons with an accuracy comparable to any of the leading ballistic systems. The equations for the interval of burning of the powder are reduced with the help of simpls approximations to the equations valid for the corresponding interval in conventional guns. Since the adiabatic law holds in this case as well as in conventional guns and rockets, an approximate solution for the interval after the powder has burned is easily obtained.

DISTRIBUTION: Copies of this report obtainable from Air Documents Division; Attn: MCIDXD

DIVISION: Ordnance and Armament (22)	SUBJECT HEADINGS:
SECTION: Guns (2)	Guns, Recoilless - Interior ballistics (47476.7)
ATI SHEET NO.: R-22-2-11	